University of Sargodha

ADP/BS 1st Semester Examination 2026

Subject: Mathematics **Paper: Vector and Tensor Analysis (MATH-5103)**

Time Allowed: 02:30 Hours **Maximum Marks: 60**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*12)

i. Define vector triple product.

ii. Define Gradient of a scalar point function.

iii. Find scalar triple product of $\vec{a} = 3\hat{\imath} - \hat{\jmath} + 5\hat{k}$, $\vec{b} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$, $\vec{c} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$.

iv. If $\varphi = \varphi(x, y, z)$ is a differentiable function, show that $= \nabla\varphi . dr$.

v. Evaluate $\vec{a} \times (\vec{b} \times \vec{c})$ if $\vec{a} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{c} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

vi. Prove that $\nabla \times \vec{r} = \mathbf{0}$.

vii. Find $\nabla^2\varphi$ if $\varphi = 2x^3y^2z^4$.

viii. Define coordinate surfaces for spherical polar coordinates.

ix. Define position vector in cylinderical coordinate system.

x. Write expression for gradient in cylinderical polar coordinates.

xi. Define Alternating Symbol.

xii. Define symmetric tensor.

Subjective Part (3*12)

Q.2. (a) Prove that $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar.

(b) If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, then find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, where $\vec{b}$ and $\vec{c}$ being non-parallel.

Q.3. (a) If $\nabla\varphi = \frac{\vec{r}}{r^5}$. Show that $\varphi(r) = \frac{1}{3}\left(1 - \frac{1}{r^3}\right)$ at $\varphi(1) = 0$.

(b) Find directional derivative of $\varphi = 4xz^3 - 3xyz^2$ at $(2, -1, 1)$ along z-axis.

Q.4. (a) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.

(b) Find the divergence of $\vec{F}$ where $\vec{F} = \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{(x^2+y^2+z^2)^{3/2}}$

Q.5. (a) Prove the orthogonality of spherical coordinate system.

(b) Derive relation for gradient and Laplacian in curvilinear coordinates.

Q.6. (a) Prove that the Alternating Symbol ϵ_{ijk} is a Cartesian tensor of rank 3.

(b) Find the symmetric and anti-symmetric parts of the tensor $\begin{bmatrix} 3 & 2 & -7 \\ -4 & 1 & 4 \\ 8 & 2 & 5 \end{bmatrix}$.